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Bond-slip parameter estimation in fiber reinforced concrete at failure using inverse stochastic model

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A B S T R A C T

Fiber reinforced concrete is an emerging material in civil engineering with many desirable properties. However, material model for this material has not been yet firmly established. Failure model has a particular property: fibers do not break in failure, they got pulled-out of the concrete matrix. Accordingly, the bond between the fiber and the matrix (bond-slip law) strongly influences material behavior in failure. In attempts to experimentally determine the bond-slip law rather large spread in results becomes evident. This variance in results can be simulated by replacing the deterministic model with the stochastic description based on the idea from the fiber bundle model. Stochastic parameters are difficult to guess so mathematical procedure based on inverse analysis has been devised. The inverse model relies on the non-linear least squares approach combined with order statistics. This novel procedure successfully extracts stochastic bond-slip parameters from experimental results.

1. Introduction

Fiber reinforced concrete (FRC) consists of concrete matrix with embedded steel (or some other) fibers. It is a rather new type of concrete but with an ever increasing use in civil engineering. By ‘new’, we mean there are no established procedures for design of structures made of FRC that would meet all the desired criteria needed for practical application of FRC [1]. The “classic” reinforced concrete contains a small number of (steel) bars and numerical model comprises model for concrete and model for steel combined in some way. On the other hand, FRC contains so many fibers that they could not be tracked individually although that could be useful for investigation of the influence of some parameters of the model [2,3]. In order to analyse FRC as a “bulk” material (which is the only approach useful for practical design) one needs either some homogenisation procedure [4] or a statistical approach.There are several approaches that address the problem; one could apply the conforming finite element model where fibers are discrete and are placed along finite element edges, like in [5]. If one would like to avoid continuum models, lattice formulation could be used instead of finite elements [6]. Anyway, the conforming approach results with a huge model that is very demanding on computer resources. On the other hand, one could apply the fiber bundle model for composites, like in [7]. Both approaches assume that the material sample comprises rather small constituents (matrix elements and fibers) that have some local properties, and under experimentation, the sample exhibits some global properties. The goal of the numerical model is connecting those local and global properties into a useful model. Achieving usefulness requires adjusting the local parameters to match the global results. Ad hoc approach is based on parametric analysis and material parameters with the most convenient results being chosen as model parameters. Besides many limitations, the result of such an approach is difficult to generalise because the parameter determination problem is not formulated as

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Fig. 1. Three-point bending of a FRC beam.

a global optimisation problem, so there is no clear overview of the relation between local and global extremes in the parameter space.

In order to better understand a probabilistic (stochastic) model it is useful to establish a relationship with a "classic" (deterministic) model of concrete. In [5] the microplane material model combined with FEM has been used to model experiments involving FRC. In the microplane model one uses an exponential curve to describe the softening stress-strain relation in the damage tensor, see [8]. Although that curve is deterministic one can obtain a similar curve using stochastic model (see Fig. 8). Stochastic material models have been applied to explain certain material behaviour that could not be described with deterministic models [9,10], e.g., spreading of experimental results. Stochastic material model is especially important for FRC since one cannot keep track of all the fibers in the material under analysis. Due to the stochastic nature of the model, in each calculation one gets slightly different stress-strain law but all the experiments have one mean and it is the mean that is of interest. We assume normal distribution of stochastic parameters that is characterised with its mean and variance. Choice of those parameters enables control (in a probabilistic sense) of the lower and the upper limit of stress-strain variations. This permits performing Monte Carlo experiments that provide realistic results directly comparable with laboratory experiments.

Two main characteristics of FRC are the fiber distribution and the bond-slip law between matrix and fibers. In [5] there is a description of influence of the bond-slip law on FRC mechanical properties. Description of some probabilistic properties of the fiber distribution and orientation in FRC is presented in [11]. In this paper the focus is on the probabilistic description of the bond-slip law between fiber and concrete matrix. This law is especially important in failure description of FRC since it has been experimentally observed that in failure fibers do not break but get pulled-out instead, Fig. 1. That characteristic is pertinent to FRC with steel fibers, in other fiber composites fibers typically break [12]. Determination of the bond-slip law between fiber and concrete is subject of the paper. That bond can be adequately described using force - displacement relation obtained from experiments and numerical model has to be able to mimic that experimentally obtained relation. Normalisation of the force-displacement relation would produce stress-strain behaviour but that is not what one normally gets from the experimental tension device. As stated in [5] determination of the bond-slip properties from experiments is difficult and time consuming. Here we are developing a procedure that would make that task simpler are more accurate.

In this work we are applying fiber bundle model that is intrinsically stochastic to describe the fiber bond in FRC and an inverse model based on order statistics for parameter determination.

Fiber bundle model [7] is intrinsically stochastic where simple constitutive elements (fibers) having stochastic properties contributing to the global material model consisting of a large number of simple elements. The model was developed by Daniels [13] in 1945 and enhanced later by numerous authors, e.g., [14–16]. More advanced version of the model that permits description of very complex behaviour is the hierarchical fiber bundle model [17,18].

The main novelties in the paper are introduction of statistical parameters with clear engineering meaning and decoupling of stochastic and deterministic model parameters so that an inverse procedure could be applied in parameter identification. Successful estimation of model parameters from experimentally obtained force - displacement curve confirms the usability of the proposed model.

2. Experimental analysis

The characteristic failure of the FRC in bending is presented in Fig. 1.

Note that in failure fibers do not break but get pulled-out; this is the behaviour pertinent to the FRC. Determination of the bond-slip law from bending has been replaced with pull-out tests on specially prepared samples. A sample consists of a concrete cube of

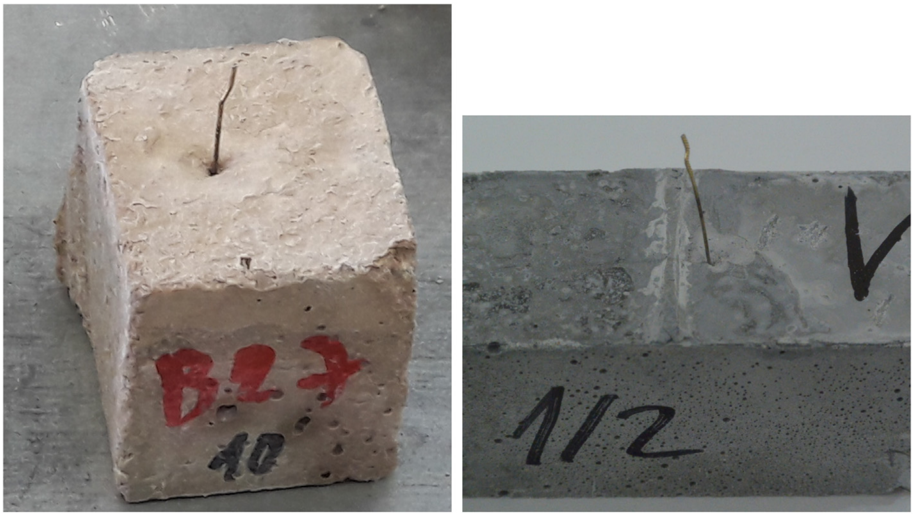

Fig. 2. Concrete samples for fiber for pull-out, left) old sample - fiber inserted after casting, right) new sample - fiber inserted before casting.

about 4 × 4 × 4 cm in size and a fiber standing out of it, see Fig. 2. Fig. 2 presents concrete sample with fibers standing out for pulling (not all embedment depths are equal). In Fig. 2 visible is the significance of proper fiber insertion into the concrete block. In the old block (left) insertion was manual after the concrete was cast and in the new block (right) fiber has been inserted prior to concrete casting. It has to be mentioned that the insertion of a steel fiber is a delicate procedure that has to be done in a manner that does not disturb the concrete specimen. Manual insertion is not acceptable since it introduces additional variance into results and makes them unusable (as we have discovered first-hand).

Fig. 3 shows the fiber pull-out experiment. The purpose of the experiment was obtaining the bond-slip relation for fibers in concrete as that governs the behaviour of the bending experiment in Fig. 1. The bond-slip relation could be deduced from the force - displacement diagram produced in the pull-out experiment (in Fig. 3). The task is not so simple since sometimes fiber would break and sometimes it would slip in the machine jaws. Solution was found in careful adjustment of the grip force in the machine jaws.

Fig. 4 presents experimental results: graphs of the force - displacement relation; here are results for two samples with two different embedment lengths with outliers due to in-jaws sliding removed.

Experimentally obtained force - displacement curves are a basis for determination of model parameters using inverse analysis.

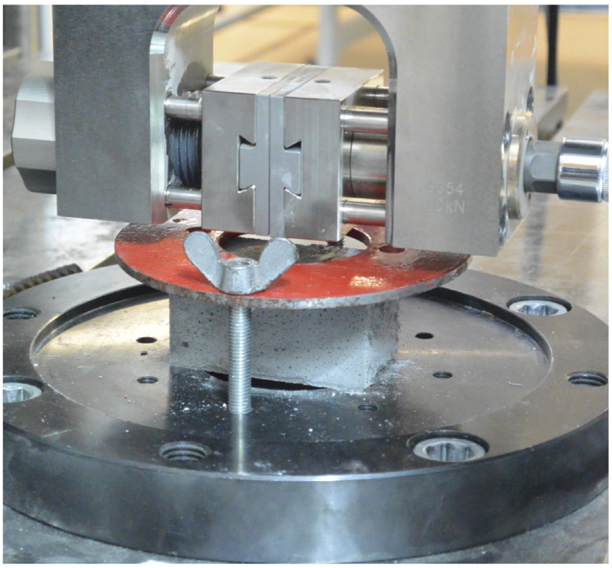

Fig. 3. Pull-out of fibers.

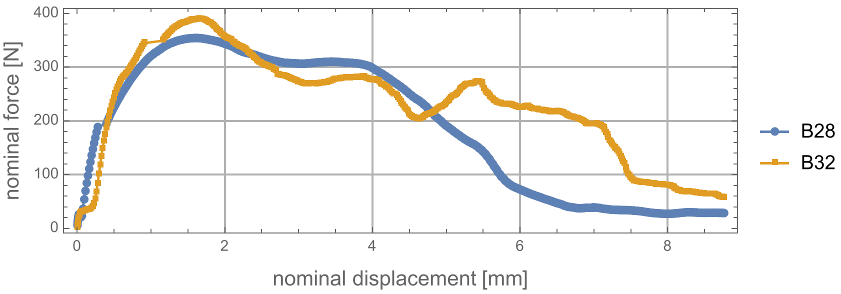

Fig. 4. Force - displacement curves for two samples (of different embedment length).

3. Material model

Material model represents the constitutive equation in analysis of structures. It (usually) relates strains with stresses in a structural model and is typically non-linear. It is called the forward model that for known loading produces displacements and stresses thus providing the desired insight into structure behaviour. Material properties and parameters (together with geometry, loading, boundary conditions) comprise the forward model. Forward models are typically deterministic meaning for the same material parameters one gets the same structure response (e.g., displacements and stresses). In nature, material parameters are stochastically distributed meaning that material models with stochastic parameters much more resemble reality. On the other side, they are much more difficult to work with as they typically require a large series of Monte Carlo experiments that are time demanding. Moreover, parameter determination requires formulation of inverse procedures that are completely different from those used for deterministic models [19].

In this work we are applying fiber bundle model (intrinsically stochastic) to describe fiber reinforced concrete and an inverse model based on order statistics for parameter determination.

3.1. Fiber bundle model

Material parameters consists of modulus of elasticity, force threshold, displacement thresholds. All stochastic parameters are independently generated based on the normal distribution with given means and variances, using Wolfram Mathematica [20]. An example of 500 randomly generated modulus of elasticity, together with sorted version needed for order statistics later, is given in Fig. 5.

One variant of the model is described in [11] but here some new properties are introduced. In order to properly mimic the experimental force - displacement curve, plastic fibers have been inserted into the fiber bundle model making it a sort of 'two-phase' material. In addition, fiber description now has two different modulus: one for loading and one for unloading. The main reason for introduction of the unloading modulus is description of the behaviour at the interface between the steel fiber and the surrounding concrete, as described in more detail in [20].

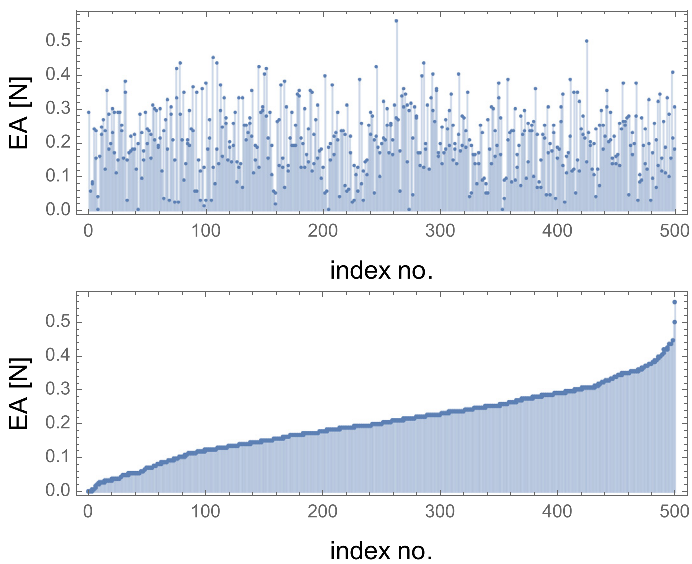

Fig. 5. Randomly generated modulus of elasticity for each fiber in the bundle, above) random function, below) sorted for order statistics.

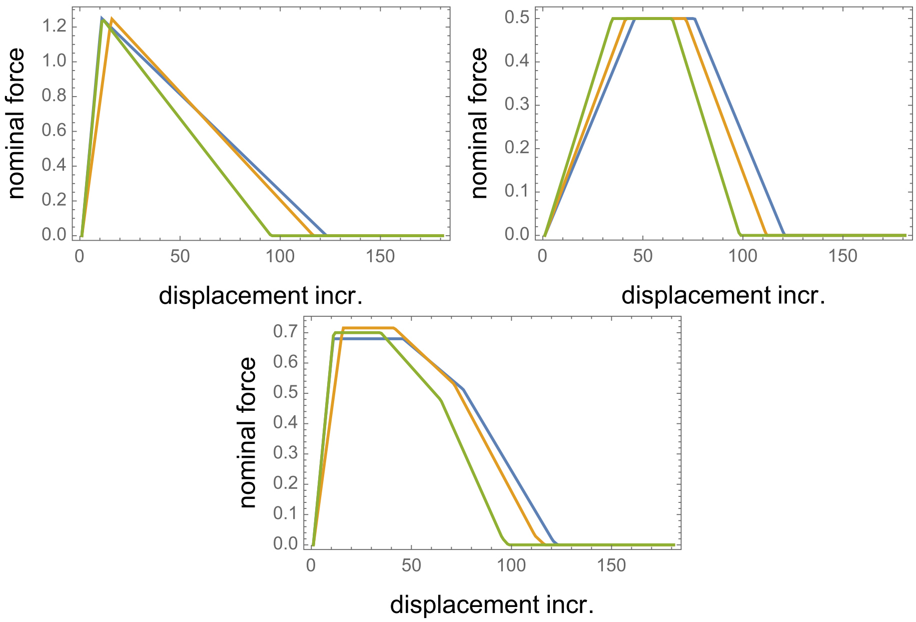

Fig. 6. Force - displacement law for three fibers: above) elastic and plastic fibers, below) combined elastic and plastic fibers.

Equation describing each elastic fiber (meaning that stochastic properties are specific for that fiber) reads

$$P_{elastic} = \begin{cases} F_u & \text{if } \delta < \delta_t \\ F_d & \text{if } \delta \geq \delta_t \wedge F_d \leq F_{t1} \wedge F_d \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$F_u = \frac{EA_{s1}\delta}{L};\ \delta_t = \frac{F_{t1}EA_{s1}}{L};_d = F_{t1} - \frac{EA_{s2}(\delta - \delta_t)}{L} \tag{2}$$

Equation describing each plastic fiber reads

$$P_{plastic} = \begin{cases} F_u & \text{if } \ \delta < \delta_t \\ F_{t2} & \text{if } \delta \geq \delta_t \wedge \delta < (\delta_t + \delta_p) \\ F_d & \text{if } \delta \geq (\delta_t + \delta_p) \wedge F_d \leq F_{t2} \wedge F_d \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$F_u = \frac{EA_{s2}\delta}{L};\ \delta_{t1} = \frac{F_{t1}EA_{s2}}{L};_d = F_{t1} - \frac{EA_{s2}(\delta - \delta_t)}{L} \tag{4}$$

In the equations above EA_s are stochastic moduli (EA_{s1} is elastic loading modulus and (EA_{s2} is elastic unloading and plastic loading and unloading modulus) and F_t is deterministic threshold force (F_{t1} is elastic threshold and F_{t2} is plastic). Plastic force additionally has plastic threshold displacement δ_p, the distance of fiber collapse after threshold force F_t has been reached. Force - displacement laws for three single elastic and plastic fibers is presented in Fig. 6 where ordinate represents the force in a fiber and abscissa the displacement. The displacement goes from 0 to 9 mm but is, for easier calculation, expressed through increments (the whole interval is divided into 180 increments, 0.05 mm each). Fig. 6 is a graphical representation of Eqs. (1)–(4) so that it is evident that each elastic and plastic fiber behaves slightly differently and the combination of elastic and plastic behaviour has slightly different response in every single fiber.

Fiber length L and fiber area A are assumed to be constant ($L = 1$, $A = 1$), it is only the modulus E that is varied; this is why we call EA the modulus although it has dimension of force. Note that when EA is assumed constant and L stochastically changes, rather different behaviour is observed, see [11].

As is visible, elastic and plastic fibers have completely different properties: different loading and unloading modulus, different force and displacement thresholds. However, within the same phase all fibers have the same force threshold.

The material is a mixture of elastic phase and plastic phase with variable ratio as given by the equation

$$P_{total} = \kappa P_{elastic} + (1 - \kappa)P_{plastic} \tag{5}$$

Total force in the fiber bundle material model is obtained through summation of the contribution of all the fibers

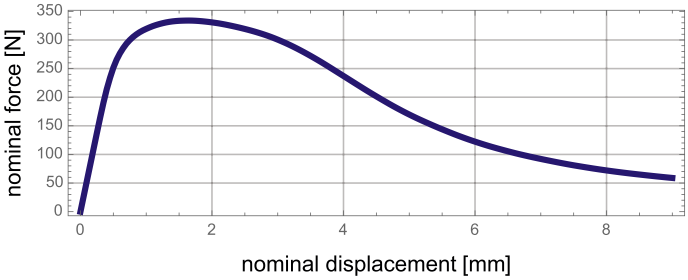

Fig. 7. Force - displacement law for the complete bundle of fibers.

$$F_{total} = \sum_{i_f=1}^{n_f} P_{total,i_f} \tag{6}$$

Here i_f is an individual fiber and n_f is the total number of fiber in the model (in our case $n_f = 500$). Contribution of all n_f fibers, i.e., its force displacement diagram is presented in Fig. 7 and it looks quite different from the contribution of an individual fiber (compare with single fiber responses in Fig. 6). Fig. 7 is a graphical representation of Eqs. (5) and (6) and was produced using initial unit values of parameters that are going to be optimised (the same values used for Fig. 6).

Force displacement law has to mimic the one acquired from experiments what is normally achieved only for certain values of parameters. Inverse procedure provides us with values of the required parameter values but before we would like to see whether it is possible to sufficiently vary the curve shape to obtain the desired one. One could use the sensitivity analysis (as in [21]) but here it is sufficient and much practical to simply use visual inspection (as in [11]).

3.2. Sensitivity analysis

Sensitivity analysis has not been performed in depth (as e.g., in [21]), only variation of parameters has been performed and graphically presented. After that, adequacy of parameters is established by visual inspection. It is evident from Figs. 8 to 10 that variation of parameters produces variations in curve shapes that are sufficient to describe results of laboratory experiments.

For convenience, in Figs. 8 to 10 displacement is expressed through increments $\Delta\delta = 0.05$ mm so the total displacement $\delta = 9.0$ mm is expressed as 180 incremental steps. Variation of the plastic yield threshold has not been presented since it has not been included as the optimisation parameter. It simply determines the length where the curve begins to decrease its value so it is rather simple to determine the value of that parameter. Variances in fiber stiffnesses have also not been set as parameters, instead, their values are fixed at 50% of the mean values of the corresponding stiffnesses.

4. Inverse model

Inverse model for non-linear problems could not be simply established from the forward procedure described in the Material model section, mainly because parameters are implicitly given in the formulation. In the case of a stochastic model the main problem is that each realisation of an experiment (numerical or physical) produces slightly different results and the inverse procedure has to take that into account [20,22]. The assumption for successful application of stochastically founded (inverse) procedures is sufficient amount of probabilistic data; in our case we usually get only a few load - displacement functions that serve for parameter determination. In this case application of the order statistics has been advantageous.

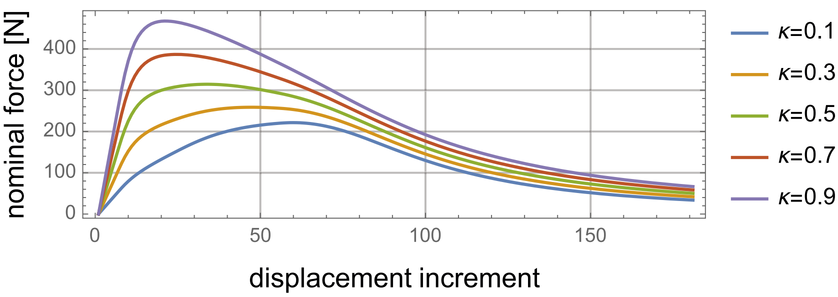

Fig. 8. Variation of the mixture ratio of elastic and plastic fibers.

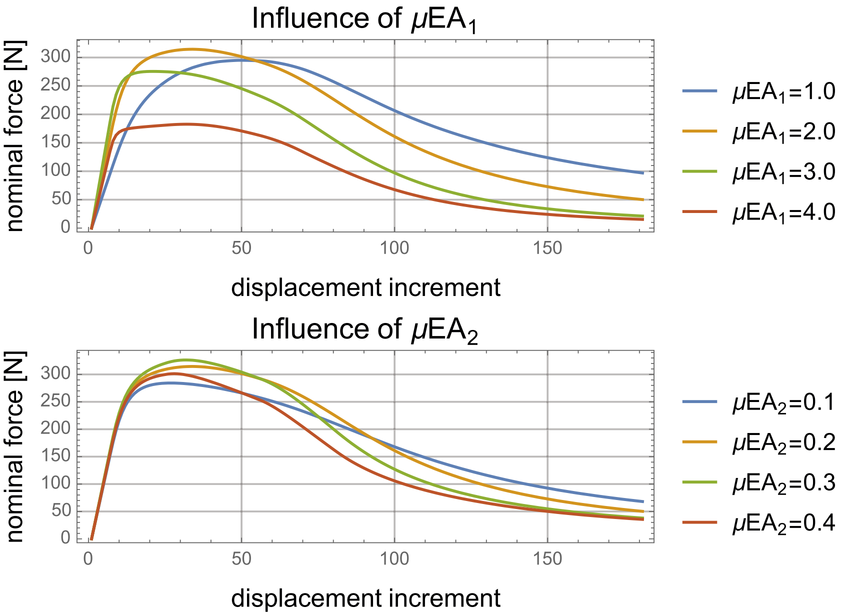

Fig. 9. Variation of the mean fiber moduli, above) for EA_1, below) for EA_2.

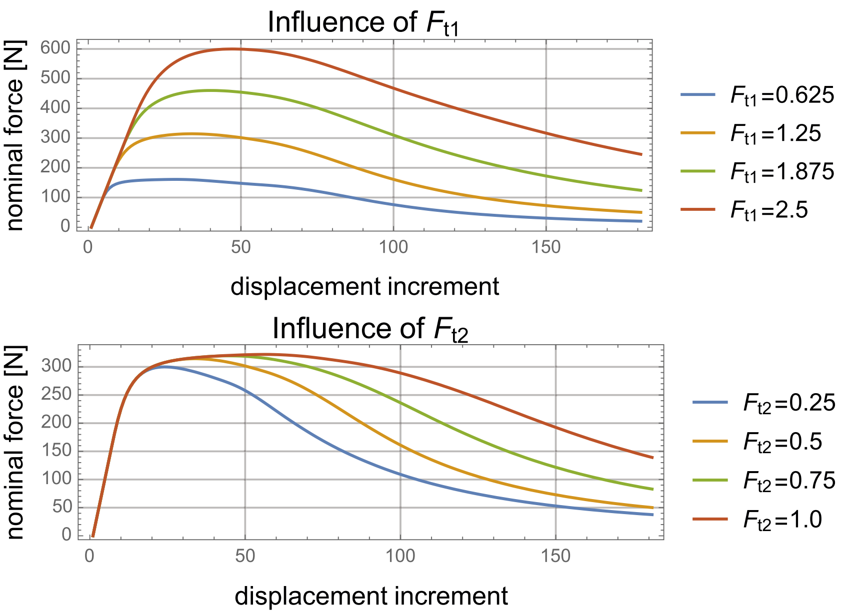

Fig. 10. Variation of the fiber force threshold, above) for Ft_1, below) for Ft_2.

4.1. Application of order statistics

Order statistics [23] and L-estimate [24] enable linear combination of data in parameter estimation. Order statistics is characterised with independent identically distributed variables X_i being arranged in ascending order $X_{1:n} \le X_{2:n} \le \cdots \le X_{n:n}$ and cumulative distribution function (CDF) is $F_{r:n} = \Pr(X_{r:n} \le x)$.

In order to present the role of order statistics in formulation of the inverse procedure we rewrite and combine Eqs. (1)–(6)

$$F_{total} = \sum_{i_f=1}^{nf} \kappa P_{el,i_f}(\delta, F_{t1}, EA_1) + (1-\kappa) P_{pl,i_f}(\delta, F_{t1}, F_{t2}, EA_1, EA_2) \tag{7}$$

Here, the probability distribution function $p(\mu, \sigma)$ is only implicitly included through stochastic parameters EA_1, EA_2. However, stochastic parameters μ, σ are not explicitly present in the force formulation and their value could not be optimised. E.g., elastic and

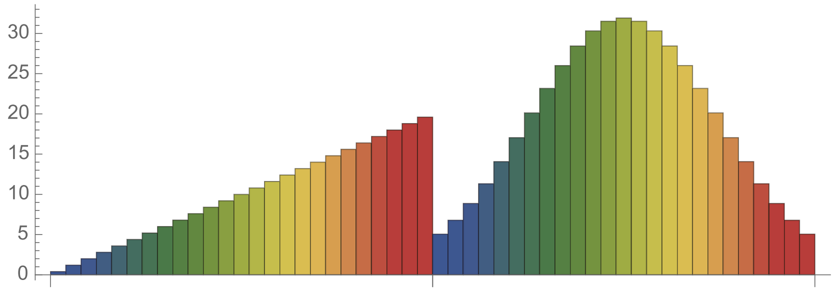

Fig. 11. Ordered EA parameters and PDF of 'bin widths'.

plastic force written with probability parameters read $P_{el,\ i_f}(\delta, F_{t1}, p(\mu_{el}, \sigma_{el}))$ and $P_{pl,\ i_f}(\delta, F_{t1}, F_{t2}, p(\mu_{pl}, \sigma_{pl}))$, respectively.

Probability distribution function $p(\mu, \sigma)$ has parameters pertinent to the chosen stochastic distribution (in our case it is the normal distribution characterised with mean μ and variance σ). In total, we have 5 parameters to determine, where 4 are stochastic parameters of functions within functions. Problem would be greatly simplified if parameters are expressed within only one function.

Simplification is achieved by replacing parameters EA_1, EA_2 with expression that includes the probability distribution function $h(\mu, \sigma)$, which could be interpreted as the bin width in the histogram representation of the pdf.

$$h1_{ib} = n_f\ \ \Delta EA_1\ f(EA1_{ib}, \mu_{el}, \sigma_{el});\ h2_{ib} = n_f\ \ \Delta EA_2\ f(EA2_{ib}, \mu_{pl}, \sigma_{pl}) \tag{8}$$

with index i_b going over the interval $[1, \cdots, n_{bin}]$ and f is a realisation of the chosen pdf $f(x) = PDF(p(\mu, \sigma), x)$. Fig. 11 shows distribution of EA parameters (equally distributed) and the corresponding 'bin widths' (stochastically distributed).

Combining of the above parameters permits the explicit formulation of fiber bundle forces (regarding the stochastic parameters) since fiber force is now a product of the stochastic and deterministic parts

$$P(\delta, F_t, p(\mu, \sigma)) = h(\mu, \sigma) Pb(\delta, F_t) \tag{9}$$

Here Pb is simply force of all fibers in the corresponding bin (bin parameters replace the single fiber parameters). The above equation is valid for bith elastic and plastic fiber forces. Now, we could rewrite the above equation explicitly including the probability distribution function parameters

$$F_{total} = \sum_{i_b=1}^{n_{bin}} \kappa\, h1_{i_b}(\mu_{el}, \sigma_{el}) Pb_{el,i_b}(\delta, F_{t1}) + (1-\kappa) h2_{i_b}(\mu_{pl}, \sigma_{pl}) Pb_{pl,i_b}(\delta, F_{t1}, F_{t2}) \tag{10}$$

We typically work with 50–150 bins. Bin influence is visible in Fig. 12 in sensitivity of the force threshold. One can conclude that the accuracy of order statistics depends on the number of bins applied in approximation of probability distribution. On the other hand, from numerical experiments it has been established that it is not of great importance since we are dealing with stochastic functions and it is only important that we conform to the mean and variance of the distribution function.

4.2. Application of non-linear least squares

Levenber-Marquart method is a variant of non-linear least squares with included regularisation parameters. Method is versatile and has a broad applicability, e.g., general information is given in [19] and an engineering application with discussion of limitations is in [25].

The method is formulated as a minimisation problem of a weighted residual (notation is as in [11]) where we first formulate a function that describes distance between measured data and data from the model

$$s_i(\mathbf{m}) = (F_u(\delta_i, \mathbf{m}) - F\delta_i); \quad \text{where } \mathbf{m} = \begin{Bmatrix} \kappa \\ \mu_{el} \\ \sigma_{el} \\ \mu_{pl} \\ \sigma_{pl} \\ d_t \end{Bmatrix} \tag{11}$$

Here, δ_i is displacement in the force - displacement diagram and $\mathbf{m}$ is vector of model parameters (κ - ratio of elastic and plastic fibers, μ, σ - mean fiber stiffness and its variance for elastic and plastic fibers, respectively, d_t - plastic threshold displacement). Index i is over sampling points on the force - displacement diagram.

Minimising the distances between measured data and data from the model results in the solution procedure for unknown model parameters

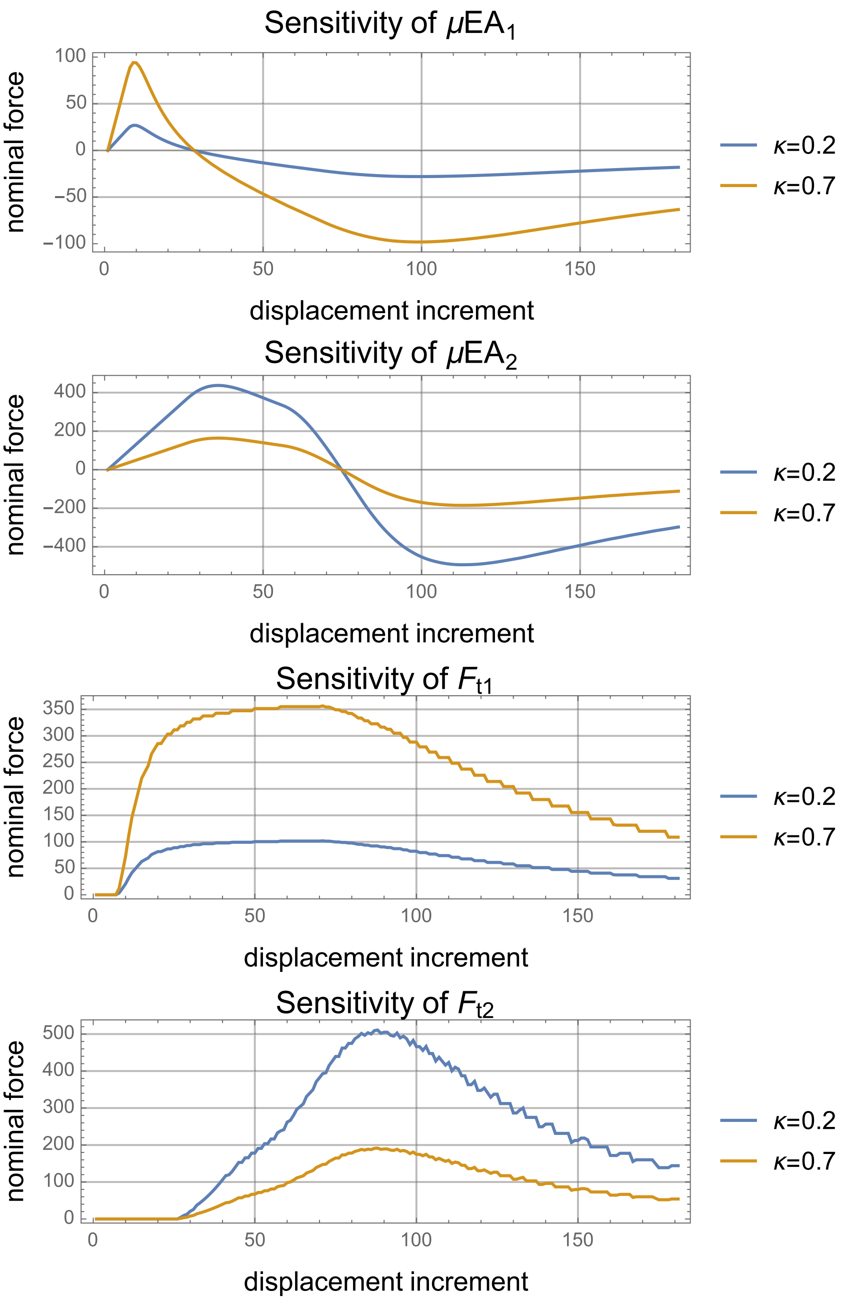

Fig. 12. Behaviour of $\partial s_1(\mathbf{m})/\partial m_1$, $\partial s_2(\mathbf{m})/\partial m_2$, $\partial s_3(\mathbf{m})/\partial m_3$, $\partial s_4(\mathbf{m})/\partial m_4$, respectively.

$$s(\mathbf{m}) = \sum_{i=1}^{np} s_i^2 = \|\mathbf{S}(\mathbf{m})\|_2^2; \quad \mathbf{S}(\mathbf{m}) = \begin{Bmatrix} s_1(\mathbf{m}) \\ \vdots \\ s_{np}(\mathbf{m}) \end{Bmatrix} \tag{12}$$

Gradient is then

$$\nabla s(\mathbf{m}) = 2\mathbf{J}(\mathbf{m})^T\mathbf{S}(\mathbf{m}); \quad \mathbf{J}(\mathbf{m}) = \begin{bmatrix} \dfrac{\partial s_1(\mathbf{m})}{\partial m_1} & \cdots & \dfrac{\partial s_1(\mathbf{m})}{\partial m_{np}} \\ \cdot & \cdots & \cdot \\ \dfrac{\partial s_{mp}(\mathbf{m})}{\partial m_1} & \cdots & \dfrac{\partial s_{np}(\mathbf{m})}{\partial m_{np}} \end{bmatrix} \tag{13}$$

We see that the Jacobian is square [$mp \times np$], i.e., [number of model parameters $\times$ number of measure points]. Solving for unknown vector of parameters leads to an incremental procedure

$$(\mathbf{J}(\mathbf{m}^k)^T\mathbf{J}(\mathbf{m}^k) + \lambda\mathbf{I})\Delta\mathbf{m} = -\mathbf{J}(\mathbf{m}^k)^T\mathbf{S}(\mathbf{m}^k) \tag{14}$$

here $\mathbf{m}^k$ is the vector $\mathbf{m}$ updated in the previous iteration step k. Note that $\mathbf{J}^T\mathbf{J}$ is quadratic matrix of size $[np \times np]$ but of rank at most mp. Normally, $np > mp$ and some regularisation procedure is needed in order to invert matrix $\mathbf{J}^T\mathbf{J}$. In the Levenberg-Marquardt method that is achieved with an additional term $\lambda\mathbf{I}$. Setting $\lambda = 0$ reduces the Levenberg-Marquardt iterative procedure to Newton's method that is possible only for small number of measure points mp. Setting $\lambda =$ large leads to steepest-descent method in which case the solution step for calculation of the update for the vector of model parameters $\mathbf{m}$ reads

$$\Delta\mathbf{m} \approx -\frac{1}{\lambda}\nabla s(\mathbf{m}) = -\frac{1}{\lambda}2\mathbf{J}(\mathbf{m})^T\mathbf{S}(\mathbf{m}) \tag{15}$$

Steepest-descent iteration has been chosen for estimation of the parameters. In addition, it is a good idea to check the behaviour of elements of the Jacobian to see if one could expect problems during the iterative procedure. In Fig. 12 there is a graph of some of the principal Jacobian elements.

Sensitivities are presented for two ratios κ of the elastic and plastic fibers and displacement is expressed through incremental steps. Derivatives are calculated numerically $\partial s_i(\mathbf{m})/\partial m_i \approx (s_i(\mathbf{m} + \delta m_i) - s_i(\mathbf{m}))/\delta m_i$. Note that the model function F_u in $s_i(\mathbf{m})$ is expressed using the order statistics that represents a numerical approximation of a statistical distribution so analytical derivatives are not possible. Functions in $s_i(\mathbf{m})$ are taken from order statistics model so steps are visible on the curves; step size depends on the number of bins taken in the order statistics model.

5. Numerical example

Numerical analysis consists of determination of parameters of the numerical model so that the force - displacement diagram from the numerical model corresponds to the experimental one. By correspondence we mean that after a series of numerical experiments the mean curve values are within some prescribed engineering tolerance and that both variances are of the similar magnitude. All examples have been calculated using [26]. Fibers in the example have length of 30 mm, diameter 0.5 mm and embedment length is 15 mm. Concrete block is 40 × 40 × 40 mm from aggregate 0–4 mm.

Parameter space is in this example four ($mp = 4$), we estimate mean stiffness for elastic and plastic fibers μEA_{s1}, μEA_{s2} and the corresponding force thresholds F_{t1}, F_{t2}. Note that μEA_{s1}, μEA_{s2} are stochastic and F_{t1}, F_{t2} are deterministic parameters.

In Fig. 13 circles represent points where values are compared, in this case number of measurement points is ten ($np = 10$).

Convergence of parameters is visible from the Table 1.

Convergence is very good as could have been expected from the behaviour of elements of the Jacobian matrix presented in Fig. 13.

Initial values of parameters have been chosen using preliminary analysis: goal function has been set for two by two parameters, each parameter having a list of values. Calculation is performed and an array of values is obtained; graphical representation of these values gives indicative value of the optimum. Real problem involves combination of five parameters but this two by two investigation can be graphically presented and is quite satisfactory for initial parameter values. Fig. 14 illustrates the procedure for mutual influence of $\mu EA2$ and $\sigma EA2$.

Coordinate values in Fig. 14 represent indexes in vectors of parameter values. When dealing with a series of experiments, this procedure is typically required only once.

6. Discussion

Here we have determined parameters that model the behaviour of steel fiber B28 with 28 mm embedment depth; the same could be done for the fiber B32 with 32 mm embedment depth. The final goal is to explain and model the variety of experimental results

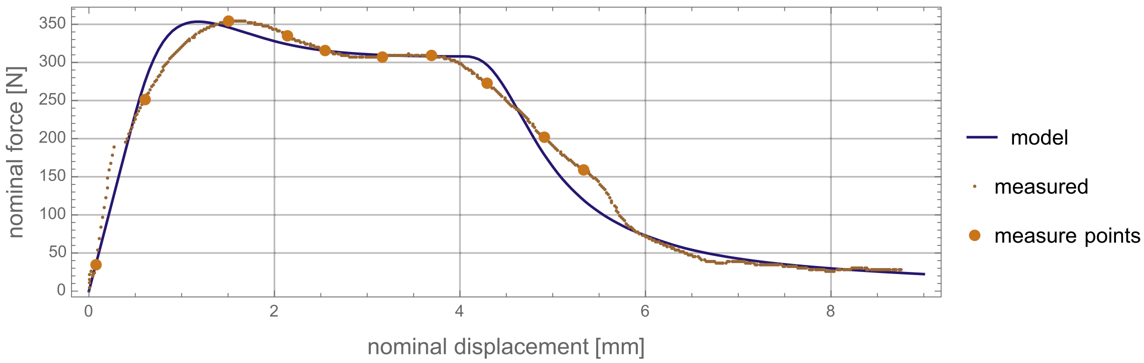

Fig. 13. Comparison of experimental and model force-displacement laws.

Table 1
Convergence properties of parameters.

Parameter	0	1	2	3	4	5	6	7
μEA_1	1.2	1.557	1.7322	1.769	1.774	1.7742	1.77425	1.77426
μEA_1	0.12	0.216	0.2239	0.2243	0.22438	0.22438	0.22438	0.22438
F_{t1}	1.0	1.285	1.3204	1.3237	1.32401	1.32404	1.32404	1.32404
F_{t2}	1.0	0.712	0.5843	0.56482	0.563632	0.563583	0.56358	0.56358

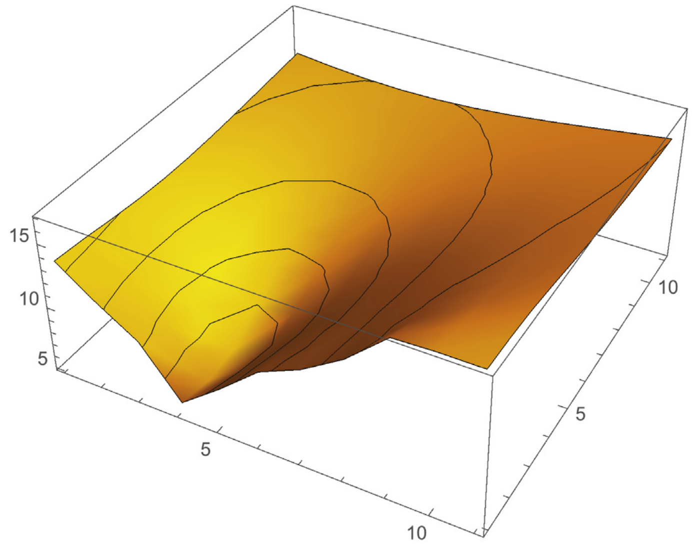

Fig. 14. Goal function for combination of parameters $\mu EA2$ and $\sigma EA2$.

obtained for pull-out of different fibers. In order to be able to do so we need more experiments with systematic variation of the material property we are interested in, in our case that would mean pull-out tests for several additional embedment depths. That analysis is in preparation and more experiments are under way. Interpretation of those new results would require reorganisation of the model since embedment depth and stiffness are in correlation.

7. Conclusion

The first part of the paper introduces a fiber bundle model as a stochastic model for description of the bond-slip behaviour between fiber and concrete during the pull-out (of the fiber). Bond-slip behaviour of the fiber is one great importance in description of the failure of fiber reinforced concrete. In order to estimate the parameter values for the model, inverse model has been formulated. Order statistics has enabled an inverse formulation of the stochastic model and the Levenberg-Marquardt iterative procedure has been used in the solution process. The inverse formulation and the solution procedure comprise a method for determination of stochastic model parameters from experimental results. It has been shown that there is no need for large number of experiments (as far as this numerical method is regarded) and one can successfully estimate both stochastic and deterministic parameters at the same time.

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