A Simple Model for Inverse Estimation from Three-point Bending Tests

Kožar, Ivica; Sulovsky, Tea; Plovanić, Marina

Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 47 (2023) 185–189

Structural Integrity Procedia

www.elsevier.com/locate/procedia

27th International Conference on Fracture and Structural Integrity (IGF27)

A Simple Model for Inverse Estimation from Three-point Bending Tests

Ivica Kožar*, Tea Sulovsky, Marina Plovanić

University of Rijeka Faculty of Civil Engineering

Abstract

The three-point bending test is used to determine the bending stiffness of materials and is a common task in civil engineering. Laboratory experiments provide us with a large amount of data, especially if the test is continued after peak loading. This allows us to determine additional material properties beyond those required by the standards. In this case, estimating properties and parameters from measurements becomes increasingly important. It is necessary to link the measured data to a mathematical model that must make the appropriate connections between the data and the model parameters. The novel numerical model contains a parameter that is determined using an inverse analysis, which is described in this paper. The validity of the model is confirmed by comparing the measured data and the calculated values.

Keywords: fiber reinforced concrete; three-point bending test; parameter estimation, inverse model;

1. Introduction

In our previous work Kožar et al. (2021a), Kožar et al. (2021b), and Kožar et al. (2022), we developed a beam model capable of mimicking laboratory experiments on three-point bending. These models are deterministic, while a stochastic model to describe fiber pullout experiments is developed in Kožar et al. (2018) and Kožar et al. (2019) and Ibrahimbegovic et al. (2020).

* Corresponding author.
E-mail address: ivica.kozar@gradri.uniri.hr

10.1016/j.prostr.2023.07.010

Nomenclature	
b	crack propagation parameter
f_k	mid-span angle tangent - parameter for post-crack bending
α	mid-span kink angle
τ	pseudo-time index

Some inverse methods for material parameter extraction are presented in Menke(2012). The extension of the analysis to fiber-reinforced concrete is presented in Rukavina et al. (2019). The evolution of cracks in brittle and quasi-brittle materials and the associated failure is presented in Mlikota et al. (2021), Liović et al. (2021), Vukelić et al. (2021), Pastorčić et al. (2019). All calculations are performed in Wolfram Mathematica Wolfram Mathematica (2022).

2. Experimental data

The original data logs include 17000 to 25000 records that were processed using moving average to reduce the number of points and speed up the analysis. Comparison of the original data with the averaged data shows that no important data were lost, i.e., all load peaks and inflection points were retained.

The experimental CMOD (crack mouth opening displacement) data for three specimens are shown in Fig. 1 in pseudo-time and as the displacement history. Fig. 2 shows the force versus displacement and the force versus CMOD.

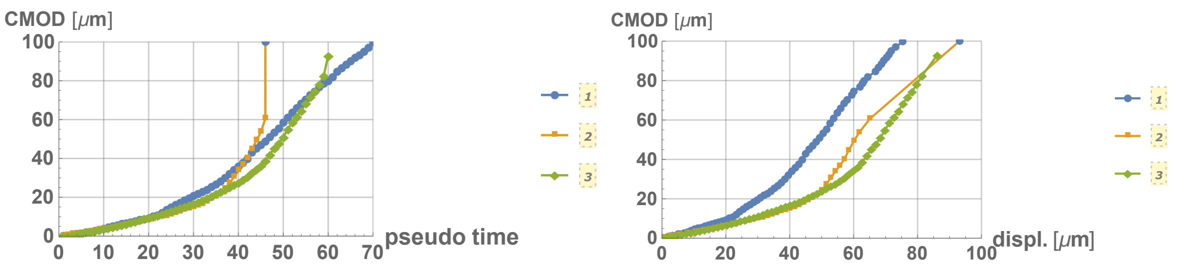

Fig. 1. (a) CMOD inpseudo-time; (b) CMOD vs. vert.displ.

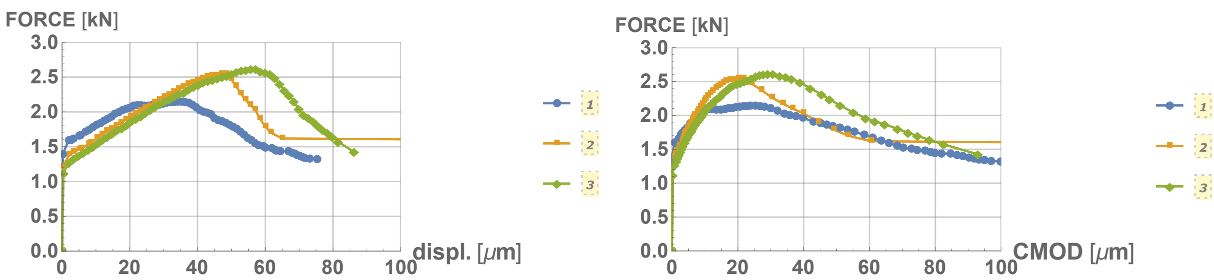

Fig. 2. (a) force vs. vert.displ.; (b) force vs. CMOD.

For use in the inverse model, the data from Fig.2 were averaged so that each measured value is represented with only one (mean) curve.

3. Three -point bending model

The mathematical model of the beam is represented by Equation 1, which is a second-order differential equation with the appropriate boundary conditions.

$$\frac{dy(x)^2}{dx^2} + \frac{M(x)}{EI(x)} = 0 \quad with\ b.c. \quad x = \frac{L}{2} \rightarrow \frac{dy}{dx} = f_k \tag{1}$$

Here $EI(x) = EI_0\,(1 - b\,x)$ and EI_0 is the full height of the beam.

We have assumed a trapezoidal shape of half of the beam and the parameter 'b' describes the reduction of the cross section in the middle of the beam. It should be in the interval [0,2], where b=0 stands for an undamaged beam and b=2 for a completely damaged beam, where the height of the middle cross-section is reduced to one point. The parameter 'f_k' represents the angle at the centre of the beam, e.g. it is zero for an undamaged beam. The parameter 'b' represents the energy due to bending and the parameter 'f_k' represents the energy due to beam rotation. It is important to note that the parameter 'f_k' is defined only as a boundary condition, so its determination by the inverse method is not straightforward and we use an a posteriori correction.

The closed- form solution of equation 1 does not converge to the simple beam solution when b=0. Consequently, we have formed the solution function y(x) as a composite function, one solution for $|b| \leq 0.0001$ and the other for all the others.

4. Inverse procedure

The numerical procedure is to solve two nonlinear equations with two unknown parameters, 'b' and 'f_k'. We have one equation, Eq. 1. The problem is to formulate the second equation. In our previous work in Kožar et al. (2021a), Kožar et al. (2021b), and Kožar et al. (2022), we used a system of two equations. Here we use a complete set of recorded data (force, displacement, CMOD), so we could work only with Equation 1. The parameter 'f_k' is considered proportional to the recorded CMOD and only 'b' needs to be determined.

Using Newton's method we solve the equation

$$y(x, b, f_k, P_m) = d_m \rightarrow x = \frac{L}{2} \tag{2}$$

for unknown 'b' and 'f_k'. Here 'P_m' and 'd_m' are data measured in the experiment but scaled by a suitable constant proportional to the value EI_0 from Eq.1. In the end, we obtain a set of solutions shown in Fig.3

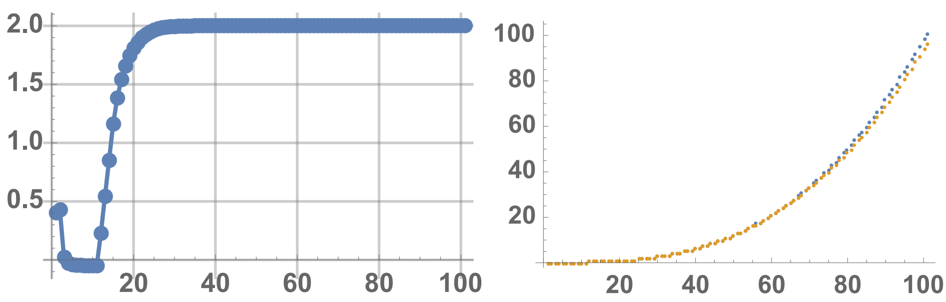

Fig. 3 (a) parameters 'b' in pseudo-time; (b) reconstruction of 'f_k' from 'b'.

In Fig.3 we see the parameters 'b' and 'f_k' determined from Equation 2. In Fig.3.a) we see the parameter 'b' which should be in the interval [0,2] with zeros at the beginning of the bending process. However, there are some values at the beginning of the simulation that are higher than zero due to fluctuations in the experimental data. In Fig.3.b), the parameter 'f_k' is recalculated from the parameter 'b' in the a posteriori correction.

In Fig.4 we see a comparison of the experimental and model results.

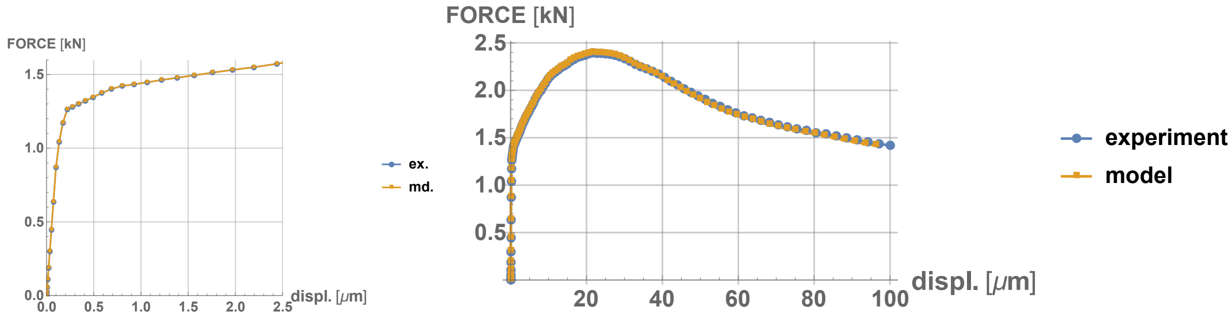

Fig. 4. Comparison of load - displ. curve experiment vs. model (a) detail; (b) total curve.

In Fig.4 we see excellent agreement between experimental and model data. It is important to include the detail in Fig.4.a) as it is affected by parameter 'b' where we see some fluctuations at the beginning of the analysis. This is probably due to the initial orientation of the experimental apparatus. Nevertheless, the model mimics the experiment very well.

The results in Fig.4 could represent a load-displacement curve or a load-CMOD curve, depending on the input data used in Equation 2, e.g., using $CMOD_m$ instead of d_m. The graphical representation of the beam bending at different stages of bending also clearly shows the influence of the parameters: at the beginning, for 'b' close to zero the beam is curved while later, when 'b' approaches the value of two, it straightens again.

As a conclusion, we note that a simple beam model with two parameters is able to mimic the three-point bending experiment. In a further step, we need to correlate our parameters with the real material and geometry properties of the experimental sample.

Acknowledgements

This work was supported by project HRZZ 7926 "Separation of parameter influence in engineering modeling and parameter identification" and project KK.01.1.1.04.0056 "Structure integrity in energy and transportation", for which we gratefully acknowledge.

References

Ibrahimbegović, A., Mathies, H.G., Karavelić, E., 2020. Reduced model of macro-scale stochastic plasticity identification by Bayesian inference in application to quasi-brittle failure of concrete. Computer Methods in Applied Mechanics and Engineering 372, 113428.

Kožar, I., Bede, N., Mrakovčić, S., Božić, Ž., 2021a. Layered model of crack growth in concrete beams in bending. Procedia Structural Integrity 31, 134-139.

Kožar, I., Bede, N., Bogdanić, A., Mrakovčić, S., 2021b. Data driven inverse stochastic models for fiber reinforced concrete. Coupled Systems Mechanics 10, 509-520.

Kožar, I., Bede, N., Mrakovčić, S., Božić, Ž., 2022. Verification of a fracture model for fiber reinforced concrete beams in bending. Engineering Failure Analysis 138, 106378.

Kožar, I., Torić Malić, N., Smolčić, Ž., Simonetti, D., 2019a. Bond-slip parameter estimation in fibre reinforced concrete at failure using inverse stochastic model. Engineering Failure Analysis 104, 84-95.

Kožar, I., Torić Malić, N., Rukavina, T., 2018. Inverse model for pullout determination of steel fibers. Coupled Systems Mechanics 7, 197-209.

Liović, D., Franulović, M., Kozak, D., 2021. Material models and mechanical properties of titanium alloys produced by selective laser melting. Procedia Structural Integrity 31, 86 – 91.

Menke, W., 2012. Geophysical Data Analysis: Discrete Inverse Theory, Academic Press Elsevier, Cambridge, Massachusetts, USA

Mlikota, M., Schmauder, S., Dogahe, K., Božić, Ž., 2021. Influence of local residual stresses on fatigue crack initiation. Procedia Structural Integrity 31, 3 – 7.

Pastorčić, D., Vukelić, G., Božić, Ž., 2019. Coil spring failure and fatigue analysis. Engineering Failure Analysis 104, 310-318.
Rukavina, T., Ibrahimbegovic, A., Kožar, I., 2019. Fiber-reinforced brittle material fracture models capable of capturing a complete set of failure modes including fiber pull-out. Comput Methods Appl Mech Eng 355, 157-192.
Vukelic, G., Vizentin, G., Božic, Ž., Rukavina, L., 2021. Failure analysis of a ruptured compressor pressure vessel. Procedia Structural Integrity 31, 28 – 32.
Wolfram Research Inc., Mathematica. 2022. URL: https://www.wolfram.com/mathematica/.